Exercise 5

If $f(x) = x + 5$ and $g(x) = x^2 - 3$, find the following.

a. $f(g(0))$ **b.** $g(f(0))$

c. $f(g(x))$ **d.** $g(f(x))$

e. $f(f(-5))$ **f.** $g(g(2))$

g. $f(f(x))$ **h.** $g(g(x))$

Solution

Evaluate each of the function compositions.

$$\begin{aligned} f(g(x)) &= f(x^2 - 3) \\ &= (x^2 - 3) + 5 \\ &= x^2 + 2 \end{aligned}$$

$$f(g(0)) = 2$$

$$\begin{aligned} g(f(x)) &= g(x + 5) \\ &= (x + 5)^2 - 3 \\ &= (x^2 + 10x + 25) - 3 \\ &= x^2 + 10x + 22 \end{aligned}$$

$$g(f(0)) = 22$$

$$\begin{aligned} f(f(x)) &= f(x + 5) \\ &= (x + 5) + 5 \\ &= x + 10 \end{aligned}$$

$$f(f(-5)) = 5$$

$$\begin{aligned} g(g(x)) &= g(x^2 - 3) \\ &= (x^2 - 3)^2 - 3 \\ &= (x^4 - 6x^2 + 9) - 3 \\ &= x^4 - 6x^2 + 6 \end{aligned}$$

$$\begin{aligned} g(g(2)) &= (2)^4 - 6(2)^2 + 6 \\ &= -2 \end{aligned}$$